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Abstract

Abductive reasoning and temporal reasoning have for the most part remained separate fields of study. Models for either tasks have made overly-simplistic assumptions about the other in order to concentrate on small highly-domain specific tasks. For example, existing abductive models have precluded any temporal information in their formulations whereas temporal models do not address abduction being only concerned with consistency issues. We present a unified approach to merging abductive and temporal reasoning. This approach properly subsumes existing abductive models as well as temporal ones and provides a precise framework for explanatory reasoning. Furthermore, effective algorithms for this new model are developed based on linear programming techniques.

1 Introduction

The need for abductive reasoning (explanation) has been demonstrated in various AI domains such as natural language understanding, medical diagnosis and planning. Formally defined as "the process of identifying the best set of assumptions to prove a given observation", several models have been proposed such as belief revision in Bayesian networks [10], cost-based abduction [8, 4] and set-covering theory [11]. Intuitively, these models attempt to provide a mechanism for reasoning the "causes from effects". Since there are typically an enormous number of causes for a given effect, the various models lay down a framework for deciding which causes are more likely to have occurred.

The notions of cause and effect implicitly require a temporal element. Yet, the existing models fail to address this. Unmistakably, the need for modeling temporal relationships has been identified in various AI applications especially those we mentioned earlier for abductive reasoning. Current abduction techniques assume a very loose temporal ordering in it's explanations. It only requires that effects should not precede causes. Hence, they are incapable of modeling more specific temporal information such as "cause A cannot precede effect B by more than 5 minutes".

On the other side of the coin, several formulations have been proposed for temporal reasoning such as Allen's interval algebra [1], point algebra [21] and temporal constraint networks [6]. Each of these models provide a rich framework for modeling temporal relationships. However, they have been used only for proving the consistency of a given set of temporal events. The issue of abduction is not addressed.

In this paper, we provide a unified approach to abduction and temporal reasoning. We were able to

- develop a formal model for representing and reasoning with both causal and temporal information permitting the rigorous analysis of the properties in this approach;
- demonstrate the natural subsumption of existing temporal and abductive models by our approach.

Thus, this provides us with a good idea of the representational power of our approach. Furthermore, we were also able to

- develop effective algorithms for finding the best temporally consistent explanation based on linear programming techniques.

Earlier work on solving abduction problems through integer linear programming were shown to be quite successful and efficient [14, 13, 3] providing us with a promising launching point for the algorithmic design at hand.

We begin our presentation with a brief review of a few of the existing abductive reasoning models in Section 2 and temporal ones in Section 3. In Section 4, we describe the formalisms of our new unified model. Finally, we present an algorithm based on integer linear programming to find the best explanation for our model in Section 5.

2 Abductive Reasoning

As we mentioned earlier, there are typically a large number of explanations (causes) to prove a given observation (effects). Unfortunately, numbered among these are explanations which we may consider to be "far-fetched". For example,

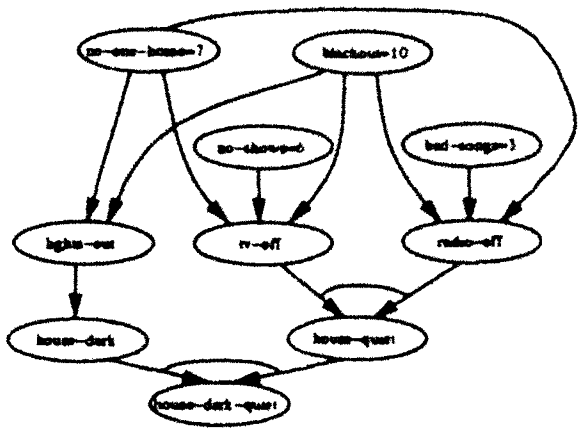

FIG. 2.1. A cost-based abduction graph.

we could conceivably explain "The dog is barking." with "Because the fire-hydrant bit it.". All the models for abduction are centered around defining a mechanism for ordering all the available explanations. In this way, it then becomes possible to identify what the "best" explanations are.

We begin by looking at the model of *cost-based abduction* developed by Charniak and Shimony [4]. Knowledge is represented in the form of a directed acyclic graph where nodes denote propositions and the arcs between the nodes represent direct logical relationships. In particular, the nodes are marked as either AND-nodes or OR-nodes. If a proposition is an AND-node, then it's truth value is the conjunction of the truth values of all it's immediate parents. Similarly, this is the case for an OR-node. Hypothesis nodes are nodes which have no parents. Evidence comes in the form of indicating that a certain subset of propositions (nodes) are true. The goal then is to find a truth assignment to the graph which is consistent to the given evidence by assuming the truth values of the various hypothesis nodes. Since there maybe numerous combinations of hypothesis node assignments to prove the evidence, a cost is attached to each hypothesis. This cost is used to reflect how likely it is to assume a certain hypothesis node. A cost can then be defined for an explanation as the sum of the costs incurred by the hypothesis nodes. The best explanation will be the explanation with minimal cost. Consider the following scenario: "John comes home from work and finds that the house is dark and quiet. He concludes that no one is home.". Figure 2.1 represents our knowledge base. Given our observation that the house is dark and quiet, we find that the minimal cost explanation is that no one is home.

Another model for abduction is *belief revision in Bayesian networks* [10].

Strictly founded in probability theory, this model also provides a nice graphical representation of the knowledge base. Again, we have a directed acyclic graph. Each node in this graph represents some random variable while arcs between the nodes denote direct conditional dependencies between the random variables. By making various instantiations to the random variables, we can model different states of the world. The conditional dependencies laid out in the network allows us to quickly compute the joint probability of a given instantiation. The probabilities can then be used to provide us with an ordering on the various world states (scenarios). The best explanation is the most probable one.

Both of these models provide a nice visualization of the knowledge base with their graphical structures. However, we feel that the cost-based approach may be much more intuitive to work with than belief revision. We can easily map conventional rule-based information into the cost-based network. Furthermore, the costs can be semantically derived as negative log probabilities on the hypothesis being true or false. In this way, Charniak and Shimony show that cost-based abduction subsumes the probabilistic model of belief revision in Bayesian networks [18] and vice versa.

As we can see, neither of these two models provides for a temporal component in their representation scheme. Other models of abduction include coherence theory [20], parsimonious covering theory [11] and weighted abduction [6, 19].

3 Temporal Reasoning

A major part of temporal reasoning has involved determining whether there is a consistent temporal ordering of a given set of events. The various events are temporally constrained between one another. For example, "The car started after Mary turned the ignition key". We have two events, "the car starting" and "the ignition key being turned" where the first event must occur "after" the second. Various temporal representations have been proposed to aid in modeling these relationships and to provide a mechanism for consistency checking.

One of the more notable models we consider is Allen's *interval algebra* [1]. Basically, there is a time interval in which each event occurs denoted by $[a, b]$ where a is the starting time point and b is the termination point. Temporal relationships between events are expressed as relations between the corresponding intervals. He identifies 13 possible relations between intervals (See Figure 3.1). In the above example, we would have the relation

$$S > K$$

where S is the starting event and K is the turning event.

Allen's formulation allows for a disjunction of relations between any two events. This can be represented in a graphical form where nodes represent events and the arcs are labeled with a disjunction of relations. The goal is to determine whether there exists an interval assignment to all the events that satisfy the

```
=                           XXX
                            YYY
<     >     precedes        XXX YYY

m     mi    meets           XXXYYY

d     di    during           XXX
                            YYYYYY
s     si    starts          XXX
                            YYYYY
f     fi    finishes          XXX
                            YYYYY
o     oi    overlaps        XXX
                             YYYYY
```

FIG. 3.1. Allen's thirteen possible relations.

disjunctive relations. If such a solution exists, then the given knowledge base is consistent.

An alternative temporal model is *temporal constraint networks* [6]. Allen's interval algebra is incapable of dealing with metric information such as *temporal distance*. For example, "Tony fell asleep 5 minutes into class" would be constrained as $T - C = 5$ where T is the time point (We are not dealing with intervals here.) that Tony fell asleep and C is when the class started. Each event is associated with a particular time point with which it occurs. A time point may be the beginning or ending of some event, as well as some neutral point of time such as 2:30pm.

Each node in a temporal constraint network represents some event. The arcs between the nodes are labeled with a disjunction of temporal distance constraints of the form

$$(a_1 \leq X_j - X_i \leq b_1) \vee \cdots \vee (a_n \leq X_j - X_i \leq b_n)$$

where the a_is and b_is are real numbers or +/- infinity. Hence, the network is consistent if there exists an assignment to the time points which satisfy the constraints.

Allen's interval algebra and the temporal constraint networks both have differing representational capabilities. As we pointed out, the interval algebra cannot model temporal distance. However, it can be shown that the temporal constraint network does not subsume the interval algebra [6].

Other models for temporal reasoning include point algebras [21], BTK [2], temporal logic [5], semi-intervals [7] and weak representations of interval algebras [9].

4 Temporal Abduction

We now formalize our unified model for abduction and temporal reasoning.

NOTATION. $\Re$ denotes the set of real numbers. $[a, b]$ such that $a, b \in \Re$ and $a \leq b$ denotes a closed interval on the real number line. Let Q denote the set of all intervals on the real numbers.[1]

If we wished to use time points as opposed to intervals, we can simply represent these points as intervals of the form $[a, a]$.

DEFINITION 4.1. *A* temporal relation *is a relation on Q.*

Allen's thirteen relations are temporal relations as well as the constraints (temporal distance, etc.) in temporal constraint networks.

DEFINITION 4.2. *A set of $\mathcal{R}$ of temporal relations is said to be* complete *if and only if given any two intervals Q_1 and Q_2 from Q, there exists a relation R in $\mathcal{R}$ such that $Q_1 R Q_2$.*

PROPOSITION 4.1. *Allen's thirteen relations form a complete set.*

NOTATION. Given a relation R, R^c denotes the transitive closure of relation R and R^{-1} denotes the inverse of R.

ϕ denotes the empty set.

DEFINITION 4.3. *A set of $\mathcal{R}$ of temporal relations is said to be* monotonic *if and only if $\overline{R} = \cup_{R \in \mathcal{R}} R$ and $\overline{R}^c \cap (\overline{R}^c)^{-1} = \phi$.*

PROPOSITION 4.2. *The subset of relations $\{<, o, s, fi, di, m\}$ from the original thirteen is a monotonic set.*

Intuitively, a monotonic set can be said to "point in only one direction." This can be used to provide a straightforward approach to modelling cause and effect in terms of the directionality of the relations. Thus, aRb can be unambiguously interpreted as *a causes b*.

DEFINITION 4.4. *A* temporal abduction problem *(abbrev.* TAP*) is a 5-tuple $W = (G, r, l, c, o)$ where*

- *$G = (V, E)$ is a graph with nodes V representing propositions or events and edges E denoting causal relationships between the nodes. We call G the* causal graph *of W.*
- *r is a mapping from V to the labels {AND, OR}. If $r(q)$ = AND, then q is also called an* AND*-node, etc.*

[1] Our approach can be easily extended to open intervals, semi-open intervals and extended intervals of the form $[a, \infty)$, etc.

- *l is a mapping from E to some non-empty collection of temporal relations. l denotes the* temporal relationships *between the nodes.*
- *c is a mapping from $V \times \{\text{true}, \text{false}\} \times \Re \times \Re$ to $\Re$ called the* cost function *for W.*
- *o is a subset of $V \times \Re \cup \{X\} \times \Re \cup \{X\}$ where X is a special symbol representing "don't care". o is called the* observations *for W.*

The causal graph represents the information for our abductive proofs. It is analogous to the graphs found in both cost-based abduction and Bayesian networks. Previously, in the two former models, an event could be proved by proving the events which are the immediate parents. This continues recursively until hypothesis nodes are reached. We augment this by further requiring certain temporal constraints to be satisfied before a parent can participate in a proof. The labels on the edges explicitly represent the temporal information.
NOTATION. For each node q in $G = (V, E)$, we define $D_q = \{p \in V | (p, q) \in E\}$ called the *parents* of q. Conversely, we define $D_q^{-1} = \{p \in V | (q, p) \in E\}$ called the *children* of q.

DEFINITION 4.5. *Given a* TAP *$W = (G, r, l, c, o)$ where $G = (V, E)$, we define an* assignment *to W to be a 3-tuple $s = (G', l', I)$ where*
- *$G' = (V', E')$ is a subgraph of G called the* solution graph.
- *l' is a mapping from E' to some set of temporal relations such that for each $e \in E'$, $l'(e) \in l(e)$.*
- *I is a mapping from V to some closed interval $[a, b]$ where $a \leq b$.*

Intuitively, $q \in V'$ if and only if q has been assigned true. Furthermore, $I(q)$ represents the time interval in which q is true.
NOTATION. Given an interval mapping $I(q) = [a, b]$, we define two projection functions $I_1(q) = a$ and $I_2(q) = b$.

DEFINITION 4.6. *An assignment $s = (G', l', I)$ for $W = (G, r, l, c, o)$ is said to be* causally sound *if and only if the following conditions hold:*
- *$q \in V'$ if there exists some $(q, a, b) \in o$.*
- *For each node $q \in V$ such that $r(q) =$ AND, if $q \in V'$, then $D_q \subseteq V'$ and $(p, q) \in E'$ for all $p \in D_q$.*
- *For each node $q \in V$ such that $r(q) =$ OR, if $q \in V'$, then $D_q \cap V' \neq \phi$ and $(p, q) \in E'$ for some $p \in D_q \cap V'$.*

Furthermore, if
- *G' is acyclic.*

then s is said to be strong causally sound.

Causal soundness simply guarantees that the propositions are directly supported/explained according to our causal information. The extra restriction of strong causal soundness guarantees an additional level support which we will return to later.

DEFINITION 4.7. *An assignment* $s = (G', l', I)$ *for* $W = (G, r, l, c, o)$ *is said to be* temporally consistent *if and only if*

- *For each pair of nodes* $p, q \in V'$ *such that* $e = (p, q) \in E'$, $I(p) R I(q)$ *where* $R = l'(e)$ *is the temporal relation.*
- *If* $(q, a, b) \in o$, *then the following conditions hold:*
  - *If* $a \neq X$, *then* $I_1(q) = a$.
  - *If* $b \neq X$, *then* $I_2(q) = b$.

Simply put, an assignment is temporally consistent if all the temporal constraints are satisfied as well as any initial conditions imposed by the observation information.

DEFINITION 4.8. *An assignment s for W is said to be an* (strong) explanation *if and only if s is (strong) causally sound and temporally consistent.*

Now, since we may have many possible explanations for a given observation, we impose an ordering in order to determine the best one.

DEFINITION 4.9. *Given an assignment* $s = (G', l', I)$ *for* $W = (G, r, l, c, o)$, *we define the* cost *of s as follows*

$$\Theta(s) = \sum_{q \in V'} c(q, \text{true}, I_1(q), I_2(q)) + \sum_{q \in V - V'} c(q, \text{false}, I_1(q), I_2(q)). \qquad (1)$$

DEFINITION 4.10. *The (strong) explanation s for W which minimizes* $\Theta(s)$ *is said to the* best (strong) explanation *for W.*

This completes our formulation of temporal abductive problems. As we can easily see, this approach merges both types of reasoning into a single theoretical framework. Let us now consider two restricted classes of TAP which provide additional properties. We denote these two classes by $\mathcal{M}_1$ and $\mathcal{M}_2$.

DEFINITION 4.11. *A* TAP $W = (G, r, l, c, o)$ *belongs in* $\mathcal{M}_1$ *if and only if G is acyclic.*

With an acyclic causal graph, we can prove the following theorem:

THEOREM 4.3. *Given a* TAP *W in* $\mathcal{M}_1$, *if s is a causally sound assignment for W, then s is also strong causally sound.*

(Proofs can be found in the Appendix.)

For the second class, we define

DEFINITION 4.12. *A* TAP $W = (G, r, l, c, o)$ *belongs in* $\mathcal{M}_2$ *if and only if the range of l is a monotonic collection of temporal relations.*

THEOREM 4.4. *Given a* TAP *W in* $\mathcal{M}_2$, *if s is a causally sound as well as a temporally consistent assignment for W, then s is also strong causally sound.*

The important property shared between these two classes is that we do not have to explicitly check for acyclicity in the solution graph given that the remaining conditions are satisfied. Cyclicity is a problem for abductive models because of the existence of "anomalous" explanations which are not properly ruled out. For example, say we have the rules that A implies B and B implies A. If we have the observation A, then we can use B to explain A. Now, we must explain B. Well, A is already true so we can use it to explain B and ad nauseum.[2]

One final note in our TAP formulation: Consider the following causal information

A and B can be used to prove C if either one of A or B precedes C.

Suppose that we have arcs from A to C and B to C such that both these arcs have two temporal labels "$<$" and "$=$". There are 4 possible labeling combinations for our two arcs. Unfortunately, we require at least one arc labeled "$<$", thus ruling out the combination with two "$=$"s. Although our present model does not directly represent this sort of temporal constraint, we can show that it can be appropriately represented by a slight modification.

We have completed our formulation of temporal abduction. We now show that existing abductive as well as temporal reasoning models are subsumed within our new framework.

For Allen's interval algebra and temporal constraint networks, the goal is to determine whether there exists a feasible solution. We can model this as a TAP problem by taking the temporal graphs and labeling the nodes as AND-nodes.

THEOREM 4.5. *Given the TAP we constructed above, an assignment s which is causally sound and temporally consistent is a feasible solution for the given temporal reasoning problem.*

For abduction, we can also prove the following theorem:

THEOREM 4.6. *The temporal abduction problem subsumes* cost-based abduction *[4],* generalized cost-based abduction *[16] and* belief revision in Bayesian networks *[10].*

Since belief revision in Bayesian networks can be transformed into cost-based abduction [18], all we need to consider is the subsumption of the cost-based approach. In the original approach, no temporal ordering is required of given nodes. We can rephrase this as allowing for any temporal ordering of the nodes. All we need to do is label the edges of the cost-based problem with all the temporal relations available. Hence, labeling with a complete set of temporal relations is sufficient. We can prove that this will provide us with the best-explanation for the original problem. (Actually, we could have identically labeled each edge in the old problem with a single temporal relation and achieve the same results. However, this seems less semantically appealing.) Hence, we have actually transformed our problem in to an $\mathcal{M}_1$ problem. Generalized cost-based abduction is proved similarly.

[2] For a detailed discussion on cyclicity in abduction, see [16].

5 Integer Linear Programming

In this paper, we have developed an effective approach for finding the best explanation in a TAP using *integer linear programming* [17]. Previous work on reducing abduction problems to integer linear programming were quite successful and efficient at determining the best one [15, 13, 3].

The transformation involves mapping the notion of propositional truth assignments into some multi-dimensional space which we will denote by $\Re^n$. A subspace of $\Re^n$ will represent "valid" truth assignments where valid includes things such as temporal consistency and causal soundness. In particular, we are interested in transforming it into a *polyhedral convex set*.[3] Such a set can be described by a collection of linear inequalities. As it turns out, these inequalities will intuitively correspond to the restrictions/constraints required in making valid truth assignments of the propositions. Finally, we would like to define a *linear energy function* such that by minimizing it over the convex set, the resulting answer will be the best explanation after we make the appropriate inverse mapping. Thus, we would have the makings of a linear constraint satisfaction problem.

Once the mapping is complete, we can then use highly efficient tools and techniques from Operations Research to solve our integer linear program. Such tools include the *Simplex method* and *Karmarkar's projective scaling algorithm* augmented with a branch and bound approach [17]. These techniques have a long history in Operations Research and are well understood.

We begin our transformation to integer linear programming as follows: Assume that the various temporal relations can be represented by a collection of linear inequalities, i.e., given temporal relation R, $I(q_1)RI(q_2)$ if and only if

$$\begin{array}{rcl} d_{1,1}a_1 + d_{1,2}b_1 + d_{1,3}a_2 + d_{1,4}b_2 & \leq & g_1 \\ \ldots & \ldots & \ldots \\ d_{k,1}a_1 + d_{k,2}b_1 + d_{k,3}a_2 + d_{k,4}b_2 & \leq & g_k \end{array}$$

where $I(q_1) = [a_1, b_1]$, $I(q_2) = [a_2, b_2]$ and $d_{i,j}, g_i$ are some constants. For example, consider Allen's "<" relation. We can represent it with the following single inequality:

$$a_2 - b_1 \leq 0 - \delta$$

where δ is some arbitrarily small but positive value.

Finally, assume that our cost-function is a linear function.

We can now proceed with transforming the abductive and temporal constraints into linear inequalities. Like values in boolean circuits, we can use numerical assignments instead of true or false. In general, we use 1 for true and 0 for false. By taking this viewpoint, we can now consider the internal

[3] "Polyhedral" refers to the fact that the boundaries of the subspace are composed of hyperplanes.

consistency as some form of mathematical formulae to be satisfied where each node is actually a variable in the equation. Our purpose is now to show how these equations can be derived and then prove that they guarantee the internal consistency required.

We begin our derivation with the simplest of the requirements. Let q be an evidence node in our TAP. Associate the variable x_q with q. Since q is an evidence node, any explanation for q must assign q to true. This can be modeled by the equation

$$x_q \quad = \quad 1. \tag{2}$$

Next, let q be an AND-node with parents D_q. We have the following: ***q* is true *iff* p is true *for all nodes p in* D_q**. Symmetrically, ***q* is false *iff there exists a p in* D_q *such that p is* false**. We can accomplish this with the following equations:

$$x_q \quad \leq \quad x_p \textit{ for each } p \in D_q \tag{3}$$

which guarantees that

1. q being true forces all p in D_q to be true, and
2. some p in D_q being false forces q to be false.

Note that at this time we are assuming that our variables may only take values of 0 or 1 although there is no upper or lower bound on the results of evaluating either side of the equation.

Finally, the OR-node can be modeled with the following equations:

$$\sum_{p \in D_q} m_{pq} \quad \geq \quad x_q \tag{4}$$

$$m_{pq} \quad \leq \quad x_p \textit{ for each } p \in D_q \tag{5}$$

where q is an OR-node with parents D_q and m_{pq} is a special marker-node indicating that p is used to explain q.[4]

We now make precise how we can transform 0-1 assignments on the real variables x_p and m_{pq} into solution graphs for the TAP and vice versa.

Let f be a 0-1 assignment to the real variables. We construct a solution graph $G'[f] = (V', E')$ for our TAP as follows:

- $p \in V'$ if and only if $x_p = 1$.
- For each $p, q \in V$ such that $r(q) =$ AND and $p \in D_q$, $(p, q) \in E'$ if and only if $x_p = x_q = 1$.
- For each $p, q \in V$ such that $r(q) =$ OR and $p \in D_q$, $(p, q) \in E'$ if and only if $m_{pq} = 1$.

Conversely, we can construct a 0-1 assignment from a solution graph as follows:

- $x_p = 1$ if and only if $p \in V'$.

[4] AND-nodes do not require marker-nodes since all the parents of such a node must be used in any explanation.

- For each $p, q \in V'$ such that $r(q) =$ AND and $p \in D_q$, $x_p = x_q = 1$ if and only if $(p, q) \in E'$.
- For each $p, q \in V'$ such that $r(q) =$ OR and $p \in D_q$, $m_{pq} = 1$ if and only if $(p, q) \in E'$.

With these transformations we can now prove the following theorem.

THEOREM 5.1. *Any 0-1 assignment f satisfies (2) - (5) if and only if $G'[f]$ satisfies the causal soundness properties in Definition 4.6.*

Together, these equations will guarantee that any feasible assignment will be causally sound which is the first step towards finding the best explanation. Next, we now define the constraints necessary for guaranteeing temporal consistency.

PROPOSITION 5.2. *Given a TAP $W = (G, r, l, c, o)$ where $G = (V, E)$, the number of variables and constraints used to guarantee causal soundness is*

- Variables

$$|V| + \sum_{\substack{q \in V \\ r(q) = \text{OR}}} |D_q| \leq |V| + |E|.$$

- Constraints

$$|o| + \sum_{\substack{q \in V \\ r(q) = \text{AND}}} |D_q| + \sum_{\substack{q \in V \\ r(q) = \text{OR}}} (1 + |D_q|) \leq |o| + |V| + |E|.$$

When a node p is used to explain node q, then one of the temporal relations specified on the edge from p to q must be satisfied. Let $\{R_1, R_2, \ldots, R_n\}$ be the relations from p to q. Associate a real variable x_{R_i} to each relation R_i indicating that the relation is satisfied. First, we construct the following constraint:

- If $r(q) =$ AND, construct

$$\sum_{i=1}^{n} x_{R_i} \geq 1 - (2 - x_p - x_q)K \tag{6}$$

where K is some arbitrarily large positive constant.

- If $r(q) =$ OR, construct

$$\sum_{i=1}^{n} x_{R_i} \geq 1 - (1 - m_{pq})K. \tag{7}$$

- For each $(q, a, b) \in o$ such that $a \neq X$,

$$a_q = a. \tag{8}$$

- For each $(q,a,b) \in o$ such that $b \neq X$,

$$b_q = b. \tag{9}$$

Next, for each relation, augment the associated linear inequalities as follows: Assume, that for relation R_i we have

$$\begin{aligned} d_{1,1}a_p + d_{1,2}b_p + d_{1,3}a_q + d_{1,4}b_q &\leq g_1 \\ \cdots \quad \cdots & \quad \cdots \\ d_{k,1}a_p + d_{k,2}b_p + d_{k,3}a_q + d_{k,4}b_q &\leq g_k \end{aligned}$$

We augment these and include them in our constraints as

$$\begin{aligned} d_{1,1}a_p + d_{1,2}b_p + d_{1,3}a_q + d_{1,4}b_q &\leq g_1 + (1 - x_{R_i})K \\ \cdots \quad \cdots & \quad \cdots \\ d_{k,1}a_p + d_{k,2}b_p + d_{k,3}a_q + d_{k,4}b_q &\leq g_k + (1 - x_{R_i})K \end{aligned} \tag{10}$$

Now, we demonstrate how we can completely transform 0-1 assignments on the real variables x_p, m_{pq}, x_R, a_p and b_p into TAP solutions.[5]

Let f be an assignment to the real variables. We construct $G' = (V', E')$, $l'(e)$ and $I(p)$ as follows:

- $p \in V'$ if and only if $x_p = 1$.
- For each $p, q \in V$ such that $r(q) =$ AND and $p \in D_q$, $(p,q) \in E'$ if and only if $x_p = x_q = 1$.
- For each $p, q \in V$ such that $r(q) =$ OR and $p \in D_q$, $(p,q) \in E'$ if and only if $m_{pq} = 1$.
- Let $\{R_1, \ldots, R_n\}$ be the relations on edge $e \in E'$. Arbitrarily choose one R_i, if any, such that $x_{R_i} = 1$ in f. Set $l'(e) = R_i$.
- $I_1(q) = a_q$.
- $I_2(q) = b_q$.

Note that we may have more than one transformation. However, this will have no effect on our results since we are mainly interested in determining whether a feasible solution exists.

Conversely, we can uniquely construct f from the temporal solution as follows:

- $x_p = 1$ if and only if $p \in V'$.
- For each $p, q \in V'$ such that $r(q) =$ AND and $p \in D_q$, $x_p = x_q = 1$ if and only if $(p,q) \in E'$.
- For each $p, q \in V'$ such that $r(q) =$ OR and $p \in D_q$, $m_{pq} = 1$ if and only if $(p,q) \in E'$.
- $x_R = 1$ if and only if $l'(e) = R$.
- $a_q = I_1(q)$.

[5] a_p and b_p are in general not assigned to 0-1. In fact, they can be assigned to anything in $\Re$.

- $b_q = l_2(q)$.

With these transformations, we can now prove the following theorem:

THEOREM 5.3. *An assignment s is temporally consistent if and only if f satisfies (6) - (10).*

PROPOSITION 5.4. *Given a* TAP $W = (G, r, l, c, o)$ *where* $G = (V, E)$, *the number of variables and constraints used to guarantee temporal consistency is*

- Variables

$$2|V| + \sum_{e \in E} |l(e)|.$$

- Constraints

$$(1 + M)|E|$$

where M is the largest number of constraints used to model a temporal relation.

We have now shown how our constraints can guarantee both causal soundness and temporal consistency. Hence, the assignments are valid explanations. To guarantee that we have a strong explanation, we must satisfy the acyclicity condition for strong causal soundness.

THEOREM 5.5. *Given a* TAP W *in either* $\mathcal{M}_1$ *or* $\mathcal{M}_2$, *an assignment s for W is a strong explanation if and only if s satisfies (2) - (5) and (6) - (10).*

Hence, we do not need to explicitly check for cyclicity in our assignment if our problem belongs in either class $\mathcal{M}_1$ or $\mathcal{M}_2$. For our more general TAP problem, it turns out we can guarantee the acyclicity condition through additional linear constraints. This can be achieved in a similar fashion as the approach described in [16] for straight abductive reasoning.

To complete our transformation to integer linear programming, all we need to do is to define an objective function to minimize. We can take the cost-function and directly use it as our objective.

THEOREM 5.6. *Given a* TAP W, *an assignment s for W is the best explanation for W if and only if s satisfies (2) - (5) and (6) - (10) and minimizes the given objective function.*

COROLLARY 5.7. *Given a* TAP W *in either* $\mathcal{M}_1$ *or* $\mathcal{M}_2$, *an assignment s for W is the best strong explanation for W if and only if s satisfies (2) - (5) and (6) - (10) and minimizes the given objective function.*

6 Conclusions

Development of abductive and temporal models of reasoning have for the most part been independent of one another. Work has mainly concentrated on small highly-domain specific problems allowing such a segregation. Previous abductive models fail to address temporal issues and assume a crude notion of time. Temporal models have provided a full representation of time but do not account for explanatory information.

We have developed a unified framework for temporal abductive reasoning. This model formally bridges the gap between abductive and temporal reasoning by merging the two areas. We have precisely defined the notion and mechanisms for explanatory reasoning taking into full account any temporal information. In particular, we have also shown that existing models of abduction and temporal reasoning such as cost-based abduction, belief revision, Allen's interval algebra and temporal constraint networks are subsumed by our model. It was shown that two restricted classes of our model, $\mathcal{M}_1$ and $\mathcal{M}_2$, are sufficient for subsumption and provide additional properties which serve to simplify our search for the best explanation.

We can transform our search for the best explanation into an integer linear programming problem allowing us to use the highly efficient tools and techniques such as Simplex from Operations Research. With this, the best explanation can be found and precisely described temporally, that is, precise values are provided for the start and end of any temporal interval.

This approach merges three fields of study together, namely, abductive reasoning, temporal reasoning and linear programming. The potential gains seem enormous in terms of representational power, reasoning capabilities, etc. One of the areas currently under study involve applying probabilistic semantics to our costs [12].

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A Proofs

THEOREM 4.3. *Given a TAP W in $\mathcal{M}_1$, if s is a causally sound assignment for W, then s is also strong causally sound.*

Proof. From Definition 4.6, $s = (G', l', I)$ is strong causally sound if it is causally sound and G' is acyclic. Given that $W = (G, r, l, c, o)$ is in $\mathcal{M}_1$, G is acyclic from Definition 4.11. Since G' is a subgraph of G, then G' is also acyclic. Thus, s is strong causally sound. □

THEOREM 4.4. *Given a TAP W in $\mathcal{M}_2$, if s is a causally sound as well as temporally consistent assignment for W, then s is also strong causally sound.*

Proof. Given that $W = (G, r, l, c, o)$ is in $\mathcal{M}_2$, let $s = (G', l', I)$ be a causally sound and temporally consistent assignment for W. Assume that G' is cyclic, that is, G' contains a directed cycle. Let $G = (V, E)$ and $G' = (V', E')$.

Without loss of generality, assume that $\{A_1, A_2, \ldots, A_n\} \in V'$ such that $(A_i, A_{i+1}) \in E'$ for $i = 1, \ldots, n-1$ and $(A_n, A_1) \in E'$. These nodes form our cycle in G'. Since $(A_i, A_{i+1}) \in E'$, $I(A_i)RI(A_{i+1})$ where $R = l'((A_i, A_{i+1}))$.

Let $R_{i,i+1} = l'((A_i, A_{i+1}))$ and $R_{n,1} = l'((A_n, A_1))$. Let

$$\overline{R} = R_{n,1} \cup R_{1,2} \cup \ldots \cup R_{n-1,n}.$$

By taking the transitive closure of $\overline{R}$, we know that

$$A_1 \overline{R}^c A_n$$

and

$$A_n \overline{R}^c A_1.$$

Hence, $\overline{R}^c \cap (\overline{R}^c)^{-1} \neq \phi$.

However, since W is in $\mathcal{M}_2$, the set of all temporal relations found in W is monotonic. Contradiction.

Therefore, G' is acyclic which implies that s is strong causally sound. □

Before we can properly prove Theorem 4.5, we provide a formulation for Allen's interval algebra and temporal constraint networks.

DEFINITION A.1. *An* interval algebra problems *(abbrev. IAP) is an ordered pair $IA = (U, k)$ where*

- *$U = (V, E)$ is an undirected graph where V is a set of events and E represents direct temporal relationships between these events.*
- *k is a mapping from E to some subset of Allen's thirteen temporal relations. In particular, this is our disjunctive labeling on the arcs.*

DEFINITION A.2. *Given an IAP $IA = (U, k)$ where $U = (V, E)$, an* assignment *s_{IA} is an ordered pair (J, k') where*

- *J maps each node in V to some closed interval.*
- *$k'(e) \in k(e)$ for all $e \in E$.*

Furthermore, we call s_{IA} *a* solution *to* IA *if and only if* $J(p)RJ(q)$ *where* $R = k'((p,q))$ *for all* $(p,q) \in E$.

DEFINITION A.3. *A* temporal constraint network *(abbrev.* TCN*) is an ordered pair* $T = (G, t)$ *where*

- $G = (V, E)$ *is a directed graph where* V *is a set of events and* E *represents direct temporal relationships between these events.*
- t *is a mapping from* E *to some collection of closed intervals representing the disjunctive inequalities between the events.*

DEFINITION A.4. *Given a* TCN $T = (U, t)$ *where* $U = (V, E)$, *an* assignment s_T *is an ordered pair* (P, t') *where*

- P *map each node in* V *to some real number.*
- $t'(e) \in t(e)$ *for all* $e \in E$.

Furthermore, we call s_T *a* solution *to* T *if and only if For all* $(p,q) \in E$, $a \leq P(q) - P(p) \leq b$ *where* $t'((p,q)) = [a,b]$.

THEOREM 4.5. *Given the* TAP *we constructed above, an assignment* s *which is causally sound and temporally consistent is a feasible solution for the given temporal reasoning problem.*

Proof. We begin with Allen's interval algebra problem. Let $IA = (U, k)$ be a IAP where $U = (V, E)$. We construct a TAP $W = (G, r, l, c, o)$ as follows:

- $G = (V, E_G)$ is a directed graph constructed from U by arbitrarily choosing an arc direction for each edge.
- $r(q) =$ AND for all $q \in V$.
- $l(e) = k(e)$ for all $e \in E$.
- $c = 0$.
- $(q, X, X) \in o$ for all $q \in V$.

Let $s_{IA} = (J, k')$ be a solution to IA. Construct an assignment $s = (G', l', I)$ for W where $G' = (V', E')$ as follows:

- $G' = G$.
- $l'(e_G) = k'(e)$ for each $e_G \in E'$ where e is the corresponding undirected arc for e in U.
- $I(q) = J(q)$ for all $q \in V$.

Now, we the following conditions on s hold:

- $q \in V'$ if there exists some $(q, a, b) \in o$.
  Clearly, this follows from our construction.
- For each node $q \in V$ such that $q \in V'$, if $r(q) =$ AND, then $D_q \subseteq V'$ and $(p,q) \in E'$ for all $p \in D_q$.
  From our construction of W, all nodes are AND-nodes. Since $G' = G$, this follows straightforwardly.

Thus, s is causally sound. Furthermore, we are guaranteed that for each pair of nodes $p, q \in V'$ such that $e = (p,q) \in E'$, $I(p)RI(q)$ where $R = l'(e)$. Therefore, s is also temporally consistent.

Now, we must prove the converse. Let $s = (G', l', I)$ where $G' = (V', E')$ be an explanation for W. Construct $s_{IA} = (J, k')$ as follows:

- $J(q) = I(q)$ for all $q \in V$.
- $k'(e) = l'(e_G)$ for all $e \in E$ where e_G is the directed edge corresponding to e in G.

From our construction, we can readily prove that s_{IA} is a solution to IA.

Hence, we can find a solution for IA by solving for some explanation in W.

Next, we must prove subsumption for temporal constraint networks. The construction is similar to Allen's interval algebra above except that we must map the time points used in TCNs to closed intervals of the form $[a, a]$.

Let $T = (G, t)$ where $G = (V, E)$ be a TCN. Construct a TAP $W = (G, r, l, c, o)$ as follows:

- $r(q) =$ AND for all $q \in V$.
- $l(e)$ will be the corresponding set of relations on closed intervals transformed from the disjunctive inequalities on the original TCN edges.
- $c = 0$.
- $(q, X, X) \in o$ for all $q \in V$.

Our proof that that all solutions for T are solutions for W an vice versa is similar in nature to the above proof for IAPs. □

THEOREM 5.1. *Any 0-1 assignment f satisfies (2) - (5) if and only if $G'[f]$ satisfies the causal soundness properties in Definition 4.6.*

Proof. Let f satisfy (2) - (5). We now prove that the solution graph $G' = (V', E')$ constructed from f satisfies the causal soundness properties in Definition 4.6.

Case 1. $q \in V'$ if there exists some $(q, a, b) \in o$.
Since $(q, a, b) \in o$, this implies from (2) that $x_q = 1$. Hence $q \in V'$ from our construction.

Case 2. For each node $q \in V$ such that $r(q) =$ AND, if $q \in V'$, then $D_q \subseteq V'$ and $(p, q) \in E'$ for all $p \in D_q$.
Assume that $q \in V'$. This implies that $x_q = 1$. From (3), $x_p = 1$ for all $p \in D_q$. From our construction, we know that $D_q \subseteq V'$. Furthermore, $(p, q) \in E'$ for all $p \in D_q$.

Case 3. For each node $q \in V$ such that $r(q) =$ OR, if $q \in V'$, then $D_q \cap V' \neq \phi$ and $(p, q) \in E'$ for some $p \in D_q \cap V'$.
Assume that $q \in V'$. This implies that $x_q = 1$. From (4), there exists some $m_{pq} = 1$ where $p \in D_q$. From our construction, $(p, q) \in E'$ which further implies that $p \in V'$. Hence, $D_q \cap V' \neq \phi$ and $(p, q) \in E'$ for some $p \in D_q \cap V'$.

Therefore, G' is causally sound.

For the converse, let G' be causally sound. Assume that f constructed from G' does not satisfy (2) - (5). This implies that one of the following constraints are violated:

Case 1. (2) : $(q, a, b) \in o$, $x_q = 1$.
Since $(q, a, b) \in o$, $q \in V'$. From our construction, $x_q = 1$. Hence, no violation.

Case 2. (3) : $r(q) = \text{AND}$, $x_q \leq x_p$ for all $p \in D_q$.
If this constraint is violated, this implies that $x_q = 1$ and $x_p = 0$ for some $p \in D_q$. From our construction, $q \in V'$ but p is not in V'. However, since G' is causally sound, $D_q \subseteq V'$. Contradiction.

Case 3. (4) : $r(q) = \text{OR}$,

$$\sum_{p \in D_q} m_{pq} \geq x_q.$$

If this constraint is violated, this implies that $x_q = 1$ and $m_{pq} = 0$ for all $p \in D_q$. From our construction, $q \in V'$ but (p, q) is not in E' for all $p \in D_q$. Contradiction.

Case 4. (5) : $r(q) = \text{OR}$, $m_{pq} \leq x_p$ for all $p \in D_q$.
If this constraint is violated, this implies that $x_p = 0$ and $m_{pq} = 1$ for some $p \in D_q$. From our construction, p is not in V' and $(p, q) \in E'$. Contradiction.

Therefore, f satisfies all the constraints. □

THEOREM 5.3. *An assignment s is temporally consistent if and only if f satisfies (6) - (10).*

Proof. Let s be temporally consistent. Assume f constructed from s does not satisfy (6) - (10). This implies that at least one of the constraints has been violated.

Case 1. (6) : $r(q) = \text{AND}$, $p \in D_q$ and

$$\sum_{i=1}^{n} x_{R_i} \geq 1 - (2 - x_p - x_q)K.$$

This constraint is violated when $x_p = x_q = 1$ and $x_{R_i} = 0$ for all i. From our construction, $e = (p, q) \in E'$. However, this implies that $l'(e) = R$ for some R on e. Thus, $x_R = 1$. Contradiction.

Case 2. (7) : $r(q) = \text{OR}$, $p \in D_q$ and

$$\sum_{i=1}^{n} x_{R_i} \geq 1 - (1 - m_{pq})K.$$

This constraint is violated when $m_{pq} = 1$ and $x_{R_i} = 0$ for all i. From our construction $e = (p, q) \in E'$. However, this implies that $l'(e) = R$ for some R on e. Thus, $x_R = 1$. Contradiction.

Case 3. (8) : $(q, a, b) \in o$, $a \neq X$ and

$$a_q = a.$$

This constraint is violated when $a_q \neq a$. From our construction, this implies that $I_1(q) \neq a$. Contradiction.

Case 4. (9) : $(q, a, b) \in o$, $b \neq X$ and

$$b_q = b.$$

This constraint is violated when $b_q \neq b$. From our construction, this implies that $I_2(q) \neq b$. Contradiction.

Case 5. (10) : Set of temporal linear inequalities.

This constraint is violated when $x_{R_i} = 1$ and a_p, b_p, a_q and b_q are set accordingly. From our construction, this implies that $(I(p), I(q))$ is not in R_i. However, $l'(e) = R_i$ and $I(p)R_iI(q)$. Contradiction.

Therefore, f satisfies the constraints.

Now, let f satisfy the constraints and construct s from f. We prove that s is temporally consistent as follows:

Case 1. If $(q, a, b) \in o$ and $a \neq X$, then $I_1(q) = a$.

Since $(q, a, b) \in o$ and $a \neq X$, (8) implies $a_q = a$. From our construction, $I_1(q) = a_q$.

Case 2. If $(q, a, b) \in o$ and $b \neq X$, then $I_2(q) = b$.

Since $(q, a, b) \in o$ and $b \neq X$, (9) implies $b_q = b$. From our construction, $I_2(q) = b_q$.

Case 3. For each pair of nodes $p, q \in V'$ such that $e = (p, q) \in E'$, $I(p)RI(q)$ where $R = l'(e)$.

Let $e = (p, q) \in E'$. This implies that $x_p = x_q = 1$ from our construction. If $r(q) =$ OR, this further implies that $m_{pq} = 1$. Either (6) or (7) implies that for some $R \in l(e)$, $x_R = 1$. Consequently, the set of inequalities associated with x_R in (10) must also be satisfied for a_p, b_p, a_q and b_q. (10) imply that $(a_p, b_p)R(a_q, b_q)$. From our construction, $l'(e) = R$ and $I(p)RI(q)$.

Therefore, s is temporally consistent. □

THEOREM 5.5. *Given a* TAP W *in either* $\mathcal{M}_1$ *or* $\mathcal{M}_2$, *an assignment* s *for* W *is a strong explanation if and only if* s *satisfies (2) - (5) and (6) - (10).*

Proof. Follows from Theorems 4.3, 4.4, 5.1 and 5.3. □

THEOREM 5.6. *Given a* TAP W, *an assignment* s *for* W *is the best explanation for* W *if and only if* s *satisfies (2) - (5) and (6) - (10) and minimizes the given objective function.*

Proof. Follows from Definitions 4.9 and 4.10 and Theorem 5.5. □

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Abductive reasoning and temporal reasoning have for the most part remained separate fields of study. Models for either tasks have made overly-simplistic assumptions about the other in order to concentrate on small highly-domain specific tasks. For example, existing abductive models have precluded any temporal information in their formulations. We present a unified approach to merging abductive and temporal reasoning. This approach properly subsumes existing abductive models as well as temporal ones and provides a precise framework for explanatory reasoning. Furthermore, effective algorithms for this new model are developed based on linear programming techniques.

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